HOME PACKAGE

FORM V SENGEREMA SECONDARY SCHOOL

ADVANCED MATHEMATICS

CALCULATING DEVICES

1. (a) The first order rate constant for the composition of ethyl iodide at initial temperature (T_1 = 600 K) is $k_1 = 6.0 \times 10^{-5} s^{-1}$. Its energy of activation (Ea) is 209000 $JmolL^{-1}$.

Using a non-programmable calculator find the rate constant (k_2) of the reaction at final

temperature ($T_2 = 700K$), if $\log\left(\frac{k_2}{k_1}\right) = \frac{Ea}{2.303R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$. Use $R = 8.314\ JmolL^{-1}K^{-1}$.

(b) Using a non-programmable scientific calculator, calculate modulus and argument of the

following complex number Z if $Z = \frac{(3-i)(2+3i)}{3+i}$.

(c) If $M^{\frac{2}{3}} = \frac{\sqrt[5]{w}}{10^{-3} \times \log_3 y + \sin^{-1} z}$, $w = 3.652 \times 10^e, y = e^{10} \times 0.00047$ and

$z = \lim_{x\to 0} \frac{\sin x}{x}$. Find the value of M to five decimal places

2. a) By using scientific calculator, compute the value of the following expressions:

i) $$\frac{\log_3\begin{vmatrix}1 & 2 & 3\\0 & 1 & 3\\2 & -1 & 5\end{vmatrix} - \left(\ln\left[\frac{3}{13}\right]^{1/3}\right) sin\left(\frac{-\pi}{6}\right)}{\int_{-2}^{3}(x-2)(x+1)(x-1)dx}$$

ii) $\frac{\sqrt{(\sqrt{19})e^{2}\ln 3}}{\sqrt{2}}$ correct to 10 significant figures.

b) By Using scientific calculator, approximate the mean and the standard deviation of the constants:

$\pi, \sqrt{2}, e, \sqrt{3}$, 1.414213, 2.718282, 3.1415, 1.732051 correct to six decimal places.

3. (a) Use a non-programmable scientific calculator to evaluate the following:

(i) $$\sum_{x=0}^{10} \frac{x\ln(x+3)}{x^2+1}$$

(ii) $$\frac{d}{dx}\left[\left(\frac{\cot 2x}{\sec x}\right)^{x+1}\right] \text{at } x = 0.2$$

(b) The volume v of a tetrahedron is given by $v=\frac{1}{6}a^3(1-\cos\theta)\sqrt{1+2\cos\theta}$ where a is the length of the edges and θ is an angle made by the edges. By completing the table below, find the volume of the tetrahedron for the given values of a and θ and write your answers correct to three decimal places.

a	θ	$\cos\theta$	$2\sqrt{\frac{1+\cos 2\theta}{2}}$	Volume (v) in cubic units
1 unit	$\frac{\pi}{12}$			
2 units	$\frac{\pi}{9}$			
3 units	$\frac{\pi}{6}$			

LINEAR PROGRAMMING

4. (a) A soft drink company has two bottling plants C_1 and C_2. Each plant produces three different soft drinks S_1, S_2 and S_3. The production of the two plants in number of bottlesper day are:

	C_1	C_2
S_1	3000	1000
S_2	1000	1000
S_3	2000	6000

A market survey indicates that during the month of April there will be demand for 24000 bottles of S_1, 16000 Bottles of S_2 and 48000 bottles of S_3. The operating cost per day for runningplants C_1 and C_2 are respectively Tsh. 600 and Tsh. 400. The company intends to find thenumber of days should the firm run each plant in April so that the production cost is minimizedwhile still meeting the market demand. Formulate the above as a linear programming model.

5. A manufacturer of a certain product has two stores, S_1 and S_2. There are 80 units of his products stored in S_1 and 70 units in S_2. Two customers, Abdullah and Ahmad orders 35 units and 60 units of the product respectively. The unit supplying cost from the stores to respective customers is given in the table below;

To / From	Abdullah	Ahmad
S_1	80/=	120/=
S_2	100/=	130/=

How should each of them be supplied in order to minimize the supplying cost?

6. A store man fills a new warehouse with two types of goods, A and B. they both come in tall boxes which cannot be stacked. A box A takes up 0.5 m^2 of floor space and costs Tsh 500. A box B takes up to 1.5 m^2 of floor space and costs Tsh 3000. The store man has up to 100 m^2 of floor space available and can spend up to Tsh 150,000 altogether. He wants to buy at least 50 boxes of A and 20 boxes of B.
   a) How many boxes of each should he buy in order to
      i) Spend all the money available and also to use as much space as possible
      ii) Use all the space for the least cost

   b) What is the cost in the second casea)(ii) above?

   A wheat flour company has factories at A and B which supply warehouses at C and D. The weekly factory capacities are 160 and 140 units respectively and the warehouse requirements are 70 and 120 units respectively. The cost of transportation of one unit of wheat flour from A to C is Sh.160 and from A to D is Sh.240. Similarly the cost of transportation of one unit of flour from B to C is Sh.200 and from B to D is Sh.260. Find how many units should be transported from each factory to each warehouse at a minimum transportation cost
   i) decimal places.

SETS

7. (a) Define the following terms:
   i. Power of set
   ii. Cardinality of a set

   (b) Show that $n(P(A)) = 2$ where A is a singleton.

   (c) A group of 82 students were surveyed, and it was found that, each of the students surveyed liked at least one of the following three fruits: namely Apricots, Bananas and Cantaloupes, 39 liked Apricots, 50 liked Banana, 39 liked Cantaloupes, 21 liked Apricots and Banana, 18 liked Banana and Cantaloupes, 19 liked Apricots and Cantaloupes and 22 liked exactly two of the three fruits. By using well labeled Venn diagram, find how many of those who surveyed liked

i. Apricots but not Banana or Cantaloupes
ii. Cantaloupes but not Banana or Apricots
iii. All of the three fruits
iv. Apricots and Cantaloupes but not Banana.

8. a) (i) What is a power set?
   (ii) Given that A = {1,2,3,4} and B = {1,4,5}. Find The power of $A\Delta B$

   b) Use the laws of algebra of sets to simplify $(A'\cap B'\cap C)\cup(B\cap C)\cup(A\cap C)$

   c) Out of 60 students in a class at a certain school, all students do at least one of the three subjects; Physics, Chemistry or Mathematics and anyone who has chosen to do Mathematics elect to do Physics as well. But no student does Mathematics and Chemistry. 50 students do Physics while 30 students do Mathematics. If 16 students do Physics and Chemistry, present the information in a Venn diagram and use it to find the number of students who do:
   i) Physics only
   ii) Chemistry only.

9. (a) Given that $A=\{x\in R; -3\le x<4\}$ and $B=\{x\in R: -2\le x<3\}$. Represent $A\cap B'$ on a number line and write the answer in set builder form.
   (b) Use the algebraic laws of sets to prove that $A\cap(A\cup B)=A$
   (c) In a town of 10,000 families it was found that 40% families buy newspaper P, 20% families buy newspaper Q, 10% families buy newspaper R, 5% families buy P and Q, 3% buy Q and R and 4% buy P and R. If 2% families buy all the three Newspapers, Find:
   (i) The number of families which buy newspaper P only.
   (ii) The number of families which buy at least one newspaper
   (iii) The number of families which buy none of P, Q and R.

FUNCTIONS

10. (a) The functions f and g are defined by $f(x)=e^x, x\in\Re, x\ge 0$ and $g(x)=x^2+1, x\in\Re$.
   Sketch the graph of $gof(x)$ and state its domain and range.

(b) If $f(x)$ has no stationary points, sketch the graph of $f(x) = \frac{x^2-4}{x^2-4x}$and state its domain and

range.

11. . a) The function f:$x \longrightarrow \frac{a}{x} + b$ is such that f(-1) = $1\frac{1}{2}$ and f(2) = 9

i) State the value of x for which f is defined

ii) Find the value of a and b

iii) Determine algebraically whether the function f is one to one or not.

b) A function f(x) is defined as:

f(x) = $\frac{3x-9}{x^2-x-2}$, determine

i) The turning points of the function

ii) The asymptotes of f(x)

iii) Sketch the graph of f(x)

12. (a) The functions f and g are defined as $f(x) = x^2 + ax + b, x \in R$ and $g(x) = 4 - 3x$, where a and b are non-zero constants. Given that $f(g(2)) = -5$ and $g(f(2)) = 29$, determine the value of a and b .

(b) Sketch $f(x) = \dfrac{x-1}{x^2-x-6}$

COORDINATE GEOMETRY 1

13. (a) i. The point R(x, y) divides a line segment joining the points A(x_1, y_1) and B(x_2, y_2) in the ratio $m\colon n$ internally. Derive the formula for the ratio theorem.

ii. Use the ratio formula in (i) to find the coordinates of the point which divides the line joining the points $(5, -4)$ and $(-3, 2)$ internally in the ratio $1\colon 2$.

(b) If the point (x, y) be equivalent from the points $(6, -1)$ and $(2, 3)$ such that $Ax + By + C = 0$, find the value of $A + B + C + 10$.

(c) Find the equation of the circle orthogonal to both the circles $x^2 + y^2 + 4x - 4y - 2 = 0$ and $x^2 + y^2 + 2x - 2y - 1 = 0$, and whose centre lie on the line $2x - 3y - 2 = 0$.

14. (a) Find the a) Without using distance formula, show that the points (-2, -1), (4, 0), (3, 3) and (3, 2) are
vertices of a parallelogram.

b) Show that the area of triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$ is $\frac{(c_1-c_2)^2}{2|m_1-m_2|}$

c) Find the equation of the circle whose centre is at the point of intersection of the two straight line: $x + y = 6$ and $2x - y - 3 = 0$ and passes through the centre of the circle $x^2 + y^2 - 2x + 2y - 10 = 0$

15. (a) Find the equation of a straight line that pass though the origin and divide the line segment joining $(4, -2)$ and $(1, 10)$ internally into the ratio $2:1$.

(b) A point $P(x, y)$ moves so that its distance from $A(7, 0)$ is equal to its distance from the y-axis. Find the locus of the point P.

(c) Show that the circles $x^2 + y^2 - 4x - 4y - 28 = 0$ and $x^2 + y^2 - 4x - 12 = 0$ touch each other externally. Hence, find the equation of the common tangent.

varies, the minimum area A will be $A = \left(16\sqrt{y} + \frac{64}{y}\right)m^2$.

LOGIC

16. (a) Use the truth table to show which is valid by saying " If an egg then a hen or if a hen then an
egg" or by saying " If an egg then a hen and if a hen then an egg"

(b) Let p represent the clouds and q represents it rains. Write a word sentence that can be represented by

(i) $\sim(p \wedge q) \rightarrow (\sim p \vee \sim q)$

(ii) $(\sim p \vee \sim q) \rightarrow [\sim(p \wedge q)]$

(c) Write the following arguments in symbolic form and test its validity.

"For candidates win, it is sufficient that he carries Dar es Salaam. He will carry Dar es Salaam only if he takes strong stand on civil rights. He will not take strong stand on civil
rights. Therefore he will not win"

17. (a) Find the compound sentence having components of P, Q, R which is true only if

exactly two of P, Q, R are true. Hence draw for a network corresponding to this

compound sentence.

(b) Let p stand for '8 is even', q stand for '4 is a factor of 5' and r stand for '3 is a prime'Write the truth value of the following statement;

(i) If 8 is even, then 4 is not a factor of 5.

(ii) Either 3 is not prime or 4 is factor of 5.

(c) Use the laws of algebra of proposition to prove that the following arguments are valid:

(i) If a Lion is a carnivorous animal then it does not eat grass. A Lion eats grass. Therefore, a Lion is not a carnivorous animal.

(ii) $p \to \sim q,\ r \to q,\ r\ \therefore \sim p$

18. a) Write the converse and inverse of thestatement:

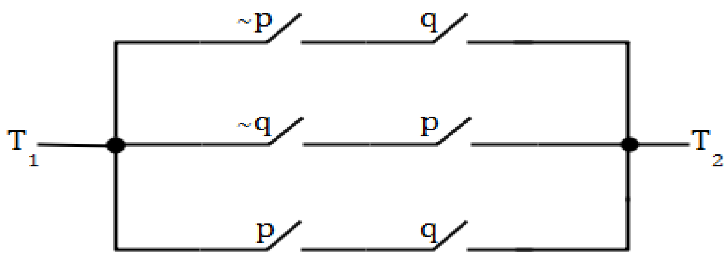
"If you score an A grade in a logic test, then I will buy you a new car"

b) Use the laws of algebraic of proposition to simplify $[p \vee (p \wedge q] \to \sim p$

c) Test the validity of the argument by using truthtable

"If I am illiterate, then I can't read and write. I cannot read but I can write. Thus, I am not illiterate"

19. (a) Draw a more simple circuit diagram than the one shown below but that can perform the same function(s).



(b) Describe the following argument in symbolic form and test its validity by using truth table:

"If he begs pardon then he will remain at school. Either he is punished or he does not remain at school. He will not be punished. Therefore, he did not beg pardon"

(c) Write the converse, inverse, and contrapositive of the statement below:

"If it rains then they postpone the football match"

TRIGONOMETRY

20. (a) Solve for x in the range 0^0 and 360^0 if $2\cos^2 x - 2\sin x\cos x + 2\sin^2 x = 1$

(b) Prove that $\dfrac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$

(c) Find the maximum and minimum value of $2\sin\theta - 5\cos\theta$ and the corresponding values of θ between 0^0 and 360^0

(d) Solve the equation $\tan^{-1}\left(\frac{1}{1+2x}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

(e) If A, B and C are angles of a triangle, prove that

$\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

21. (a) Solve the equation $4\cos x-6\sin x=5$, for values of x between 0^0 and 360^0 correct to 0.1^0.

(b) Find the general solution of $\sin 3x\cos 3x-\cos^2 2x+\frac{1}{2}=0$

(c) Solve the simultaneous equations

$\cos x+\cos y=1$, $\sec x+\sec y=4$ for $0^0<x<180^0$, $0^0<y<180^0$.

(d) If $\sec 2B-\tan 2B=a$, prove that $\tan B=\frac{1-a}{1+a}$.

(e) Prove the following identity: $\frac{\sin(n+1)A+2\sin nA+\sin(n-1)A}{\cos(n-1)A-\cos(n+1)A}=\cot\frac{A}{2}$

22. a) If $7\tan\theta+\cot\theta=5\sec\theta$ for $0°\leq\theta\leq180°$, solve the equation.

b) Show that $3\cos\theta+2\sin\theta$ can be written as $\sqrt{13}\cos(\theta-\alpha)$. Hence find the minimum and maximum values of the function by giving the corresponding values of θ between -180° and180

23. a) Solve the equation $\tan^{-1}\left(\frac{x-1}{x+2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{2}$

(b) Given that $\sin A=\frac{3}{5}$ and $\cos B=\frac{24}{25}$, where angle A is obtuse and angle B is acute, find the exact value of $\cot(A-B)$.

(c) For all values of β show that $\frac{\sin 3\beta}{\sin\beta}-\frac{\cos 3\beta}{\cos\beta}=2$

ALGEBRA

24. (a) Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x^4}\right)^6$

(b) Solve the equations below:

$$\begin{cases} \log(x+y) = 0 \\ 2\log x = \log(y+1) \end{cases}$$

(c) Prove by Mathematical Induction that $3^n - 1$ is a multiple of 2.

(d) Find the values of x if $\left|\frac{x+3}{x+1}\right| < \left|\frac{x-2}{x-3}\right|$.

(e) If α and β are the roots of the equation $2x^2 + 3x - 4 = 0$. Find the equation with the roots

$\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(f) Use synthetic division to find the value of "d" given that the polynomial

$P(x) = x^3 + dx^2 - 2dx + 4$ is divisible by $x - 1$

25. (a) Solve the simultaneous equations,

$\frac{x^2}{4} - \frac{1}{y+1} = 1, \quad \frac{1}{3(y+1)} + \frac{x}{2} = 3.$

(b) The second, third and fourth terms in the expansion of $(x+a)^n$ are $240, 720$ and 1080 respectively. Find n, x and a.

(c) For what values of x is the function $2x^2 + 5x - 3$ is negative?

(d) If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, obtain the equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$ .If, in the above equation $\alpha\beta^2 = 1$,

prove that $\alpha^3 + c^3 + abc = 0.$

(e) By using Cramer's rule solve the following system of equations:

$x + 4y + 3 = 3z$, $-10y + x + 7z = 13$ and $x - 2y + z = 3.$

26. a) If the sum of squares of the roots of equation $ax^2 + bx + c = 0$ is one, prove that $b^2 = a^2 + 2ac$

b) Using synthetic division, find the value c given that the polynomial: $P(x) = x^3 + cx^2 - 2cx + 4$ is divisible by $x - 1$

27. (a) Find the first four terms in the expansion of $\frac{1}{(2+3x)^2}$ and state the range of values of x for which the expansion is valid.

(b) Solve the system of equations by the inverse matrix method:

$$\begin{cases} p+q-r=-3 \\ 2p-3q+4r=23 \\ -3p+q-2r=-15 \end{cases}$$